

1/19/16

HW: Midterm Review #1-11

Midterm Friday 1/22

Name _____
Review

Calculus

This review sheet should NOT serve as your only review. You should review all notes and tests.

Questions 1 through 7 refer to the graph of $y = f(x)$ shown to the right.

1. $\lim_{x \rightarrow 1^-} f(x) = -\infty$

2. $\lim_{x \rightarrow 1^+} f(x) = \infty$

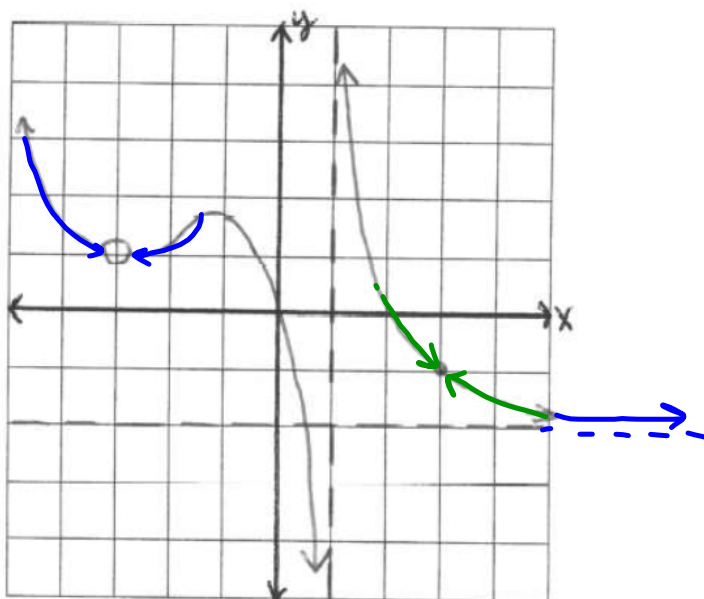
3. $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

4. $\lim_{x \rightarrow -3} f(x) = 1$

5. $\lim_{x \rightarrow 3} f(x) = -1$

6. $\lim_{x \rightarrow -\infty} f(x) = \infty$

7. $\lim_{x \rightarrow \infty} f(x) = -2$

For each of the following functions, use the definition of derivative to find $f'(x)$.

Recall: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

8. $f(x) = 2x^2 - 8x + 5$

9. $f(x) = \sqrt{3x+1}$

Find the derivative of each of the following:

10. $f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$

11. $f(x) = \sin^2(3x+1)$

12. $f(x) = \ln(\sin x)$

13. $f(x) = \ln(\sqrt{2x+3})$

14. $f(x) = \frac{e^{2x}}{x^2}$

15. $f(x) = \sqrt[4]{(x^2+5x)^3}$

16. $f(x) = \sqrt{x} \tan x$

17. $f(x) = x^3 \sec(e^{3x}-1)$

18. $f(x) = e^{\sqrt{2x}}$

19. Find the slope of the line tangent to $y = x^2 \ln(3x)$ when $x = 1$.

8) $f(x) = 2x^2 - 8x + 5$ $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$\lim_{h \rightarrow 0} \frac{2(x+h)^2 - 8(x+h) + 5 - 2x^2 + 8x - 5}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{2x^2} + 4xh + \cancel{2h^2} - \cancel{8x} - 8h + \cancel{5} - \cancel{2x^2} + \cancel{8x} - \cancel{5}}{h}$$

$$\lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 8h}{h} = \frac{h(4x + 2h - 8)}{h} = 4x + 2h - 8$$

$$h=0 \therefore 4x + 2(0) - 8$$

$$f'(x) = \boxed{4x - 8}$$

$$9) f(x) = \sqrt{3x+1}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h} \quad \left(\frac{\sqrt{3(x+h)+1} + \sqrt{3x+1}}{\sqrt{3(x+h)+1} + \sqrt{3x+1}} \right)$$

$$\frac{3(x+h)+1 - (3x+1)}{h(\sqrt{3x+3h+1} + \sqrt{3x+1})} = \frac{\cancel{3x} + \cancel{3h} + 1 - \cancel{3x} - 1}{h(\sqrt{3x+3h+1} + \sqrt{3x+1})}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{3} \cancel{h}}{\cancel{h}(\sqrt{3x+3h+1} + \sqrt{3x+1})} = \lim_{h \rightarrow 0} \frac{3}{\sqrt{3x+3h+1} + \sqrt{3x+1}}$$

Let $h=0$

$$\frac{3}{2\sqrt{3x+1}}$$

Check:

$$f(x) = \sqrt{3x+1} = (3x+1)^{1/2}$$

$$f'(x) = \frac{1}{2} (3x+1)^{-1/2} (3)$$

$$= \frac{3}{2(3x+1)^{1/2}} = \frac{3}{2\sqrt{3x+1}}$$

$$10) f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$$

$$f(x) = 5x + 2x^{\frac{1}{3}} - 3x^{-2}$$

OR

$$f'(x) = 5 + \frac{2}{3}x^{-\frac{2}{3}} + 6x^{-3}$$

$$f'(x) = 5 + \frac{2}{3\sqrt[3]{x^2}} + \frac{6}{x^3}$$

$$11) f(x) = \sin^2(3x+1) = (\sin(3x+1))^2$$

$$f'(x) = \underline{2}(\sin(3x+1)) \cdot (\cos(3x+1))(\underline{3})$$

$$= \underline{6}(\sin(3x+1))(\cos(3x+1))$$

$$12) f(x) = \ln(\sin(x))$$

$$f'(x) = \frac{1}{\sin(x)} \cdot \cos(x) = \frac{\cos(x)}{\sin(x)} = \boxed{\cot(x)}$$

$$13) f(x) = \ln(\sqrt{2x+3}) = \ln((2x+3)^{\frac{1}{2}})$$

$$f'(x) = \frac{1}{\sqrt{2x+3}} \cdot \cancel{\frac{1}{2}}(2x+3)^{\cancel{\frac{1}{2}}} \cdot \cancel{2}$$

$$= \frac{1}{\sqrt{2x+3}} \cdot (2x+3)^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2x+3}} \cdot \frac{1}{(2x+3)^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{2x+3}} \cdot \frac{1}{\sqrt{2x+3}} = \left(\frac{1}{2x+3} \right)$$

$$14) f(x) = \frac{e^{2x}}{x^2}$$

$$f'(x) = \frac{(x^2)(e^{2x})(2) - (e^{2x})(2x)}{(x^2)^2}$$

$$f'(x) = \frac{2e^{2x} \cancel{x} - 2e^{2x} \cancel{x}}{x^4}$$

$$f'(x) = \frac{2xe^{2x} - 2e^{2x}}{x^3}$$

$$15) f(x) = \sqrt[4]{(x^2+5x)^3} = (x^2+5x)^{3/4}$$

$$f'(x) = \frac{3}{4}(x^2+5x)^{-1/4} \cdot (2x+5)$$

$$= \frac{3}{4(x^2+5x)^{1/4}} \cdot \frac{2x+5}{1} = \frac{6x+15}{4\sqrt[4]{x^2+5x}}$$

16) OMIT

17) OMIT

$$18) f(x) = e^{\sqrt{2x}} = e^{(2x)^{1/2}}$$

$$f'(x) = e^{\sqrt{2x}} \cdot \left(\frac{1}{2}\right)(2x)^{-1/2} \cdot (2)$$

$$= \frac{e^{\sqrt{2x}}}{(2x)^{1/2}} = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

19) Plug $x=1$ into the derivative

$$y = x^2 \cdot \ln(3x) \quad y' = f'g + fg'$$

$$y' = (2x)(\ln(3x)) + (\cancel{x^2})\left(\frac{1}{3x}\right)(3)$$

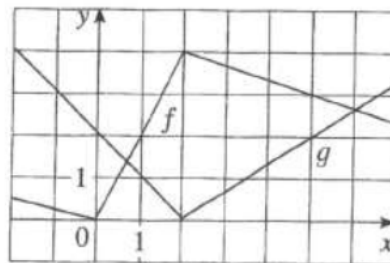
$$y' = (2x)(\ln(3x)) + x$$

Plug 1 into x $y' = (2(1))(\ln(3(1))) + 1$

$$@x=1 \rightarrow y' = 3.197$$

20. Write the equation of the line tangent to $y = 3x^2 - 2x + 1$ when $x = -1$.
21. Write the equation of the normal to $y = 5 - x^2$ when $x = 2$.
22. An object moves along a line so that its position at time t is given by $s(t) = 2t^3 - 15t^2 + 24t - 10$ where $t \geq 0$.
- What is the object's position at time $t = 3$?
 - What is the object's velocity at time $t = 3$?
 - What is the object's acceleration at time $t = 3$?
 - Is the object speeding up or slowing down at $t = 3$? Justify your response.
 - When is the object at rest?
 - When is the object moving right?
 - How far does the object travel in the first 3 seconds?
23. If f and g are the functions shown below. Let $h(x) = f(g(x))$ and $s(x) = f(x)g(x)$.

Find: $h'(1)$ and $s'(1)$



24. The following table records the values of f , f' , g , and g' at $x = 1$, $x = 2$, and $x = 3$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	2	3
2	5	4	3	4
3	0	6	-1	-2

If $n(x) = \frac{f(x)}{g(x)}$, $h(x) = f(g(x))$, find the value of each of the following: a) $n'(2)$ b) $h'(1)$

25. If $f(x) = \sqrt[3]{(x^2 - 2x - 1)^2}$, then $f'(0) =$

28. Find $\frac{dy}{dx}$ for the given curve: $x^3 + y^3 = 18y$

29. Find $\frac{dy}{dx}$ for the given curve: $x^2y - xy^2 = 4x$

30. Write the equation of the tangent to $x^2 - xy = y^2 + 1$ in the first quadrant when $y = 1$.

20) Equation we need: Slope (derivative) Point (function)

Slope: @ $x = -1$

$$y = 3x^2 - 2x + 1$$

$$y' = 6x - 2$$

$$y' = 6(-1) - 2$$

$$y' = -8 \leftarrow \text{slope}$$

Point: $x = -1$

$$y = 3(-1)^2 - 2(-1) + 1$$

$$y = 6$$

$$(-1, 6)$$

$$y - 6 = -8(x + 1)$$

21) Normal negative reciprocal slope of tangent.

Point: $x = 2$

$$y = 5 - x^2$$

$$y = 5 - 2^2$$

$$y = 1$$

$$(2, 1)$$

$$y - 1 = \frac{1}{4}(x - 2)$$

Slope: @ $x = 2$

$$y' = -2x$$

$$y' = -2(2)$$

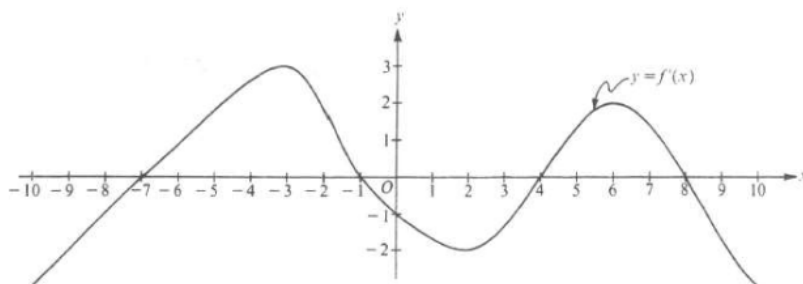
$$y' = -4 \leftarrow \text{slope of tangent}$$

$$\frac{1}{4} \leftarrow \text{slope of Normal}$$

33. Given the function $f(x) = x^4 - 4x^3$, find:

- the zeros of the function
- the critical points and the intervals of increasing and decreasing.
- Any possible inflection points and intervals of concave up or concave down.
- Finally, sketch the graph. Use your analysis from the 1st and 2nd derivative tests and the zeros you found.

34.



Note: This is the graph of the derivative of f , not the graph of f .

The figure above shows the graph of f' , the derivative of a function f . The domain of f is the set of all real numbers x such that $-10 \leq x \leq 10$.

- For what values of x does the graph of f have a horizontal tangent?
- For what values of x in the interval $(-10, 10)$ does f have a relative maximum?
Justify your answer.
- For what values of x is the graph of f concave downward?

$$22) s(t) = 2t^3 - 15t^2 + 24t - 10 \quad s(3) = -19 \quad (a)$$

$$v(t) = 6t^2 - 30t + 24 \quad v(3) = -12 \quad (b)$$

$$a(t) = 12t - 30 \quad a(3) = 6 \quad (c)$$

d) Slowing b/c $v(t)$ and $a(t)$ are not same sign

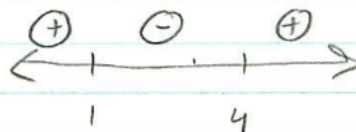
e) $v(t) = 0$

$$0 = 6t^2 - 30t + 24$$

$$0 = 6(t^2 - 5t + 4)$$

$$0 = (t-4)(t-1)$$

$$= t=4 \quad t=1$$



$$f) (0, 1) \cup (4, \infty)$$

$$g) |s(0) - s(1)| + |s(1) - s(3)|$$

$$|-10 - 1| + |1 - (-19)|$$

$$11 + 20$$

$$(31) \text{ units}$$

$$23) h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(1) = f'(g(1)) \cdot g'(1)$$

$$= f'(1) \cdot (-1)$$

$$= 2 \cdot (-1) = \boxed{-2}$$

$$\dot{s}(x) = f(x)g'(x) + f'(x)g(x)$$

$$= f(1)g'(1) + f'(1)g(1)$$

$$= (2)(-1) + (2)(1)$$

$$= -2 + 2$$

$$= \boxed{0}$$

$$24) a) r'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$r'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} = \frac{(3)(4) - (5)(4)}{3^2} = \frac{12 - 20}{9} = \boxed{\frac{-8}{9}}$$

$$\begin{aligned}
 b) \quad h'(x) &= f'(g(x)) \cdot g'(x) \\
 &= f'(g(1)) \cdot g'(1) \\
 &= f'(2) \cdot g'(1) \\
 &= 4 \cdot 3 \\
 &= \boxed{12}
 \end{aligned}$$

$$\begin{aligned}
 25) \quad f(x) &= \sqrt[3]{(x^2-2x-1)^2} = (x^2-2x-1)^{\frac{2}{3}} \\
 f'(x) &= \frac{2}{3} (x^2-2x-1)^{-\frac{1}{3}} \cdot (2x-2) \\
 &= \frac{2(2x-2)}{3 \sqrt[3]{x^2-2x-1}} = \frac{4x-4}{3 \sqrt[3]{x^2-2x-1}} \\
 f'(0) &= \frac{4(0)-4}{3 \sqrt[3]{0^2-2(0)-1}} = \frac{-4}{3(-1)} = \frac{-4}{-3} = \boxed{\frac{4}{3}}
 \end{aligned}$$

$$28) \quad x^3 + y^3 = 18y \quad 3x^2 + 3y^2 \frac{dy}{dx} = 18 \frac{dy}{dx}$$

$$3x^2 = 18 \frac{dy}{dx} - 3y^2 \frac{dy}{dx}$$

$$3x^2 = (18 - 3y^2) \frac{dy}{dx}$$

$$\frac{3x^2}{18-3y^2} = \frac{dy}{dx} = \boxed{\frac{x^2}{6-y^2}}$$

$$29) \quad x^2 y - x y^2 = 4x$$

$$x^2 \frac{dy}{dx} + 2xy - x 2y \frac{dy}{dx} + y^2 = 4$$

$$x^2 \frac{dy}{dx} + 2xy - 2xy \frac{dy}{dx} - y^2 = 4$$

$$\frac{dy}{dx} (x^2 - 2xy) = 4 - 2xy + y^2$$

$$\frac{dy}{dx} = \boxed{\frac{4 - 2xy + y^2}{x^2 - 2xy}}$$

$$30) \quad x^2 - xy = y^2 + 1 \quad \text{when } y=1$$

Point: $x^2 - x(1) = 1^2 + 1$

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x=2 \quad x=-1$$

reject
not in
QI

Point: $(2, 1)$

Slope = $2x - x \frac{dy}{dx} - y = 2y \frac{dy}{dx}$

$$2x - y = \frac{dy}{dx} (2y + x)$$

$$\frac{2x - y}{2y + x} = \frac{dy}{dx}$$

$$\frac{2(2) - 1}{2(1) + 2} = \frac{3}{4} \quad \leftarrow \text{slope @ } (2, 1)$$

$$\boxed{y - 1 = \frac{3}{4}(x - 2)}$$

33) $f(x) = x^4 - 4x^3$
 $f'(x) = 4x^3 - 12x^2$

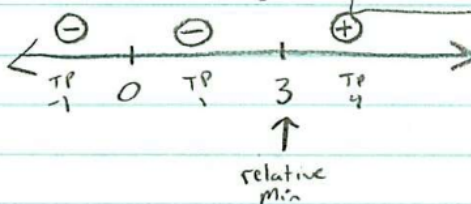
$f(x)=0$

a) $0 = x^4 - 4x^3$
 $0 = x^3(x-4)$
 $x=0 \quad x=4$

b) $f'(x)=0$
 $0 = 4x^3 - 12x^2$
 $0 = 4x^2(x-3)$
 $x=0 \quad x=3$

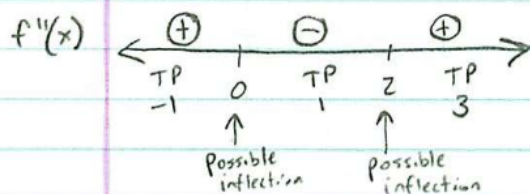
Increasing:
 $(3, \infty)$

Decreasing:
 $(-\infty, 0) \cup (0, 3)$



$f''(x)=0$

c) $f''(x) = 12x^2 - 24x$
 $0 = 12x(x-2)$
 $x=0 \quad x=2$

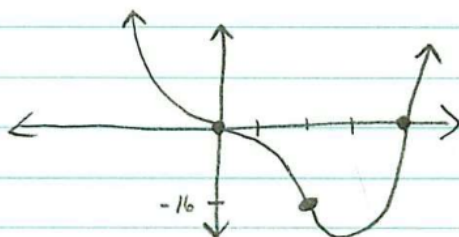


Concave Up: $(-\infty, 0) \cup (2, \infty)$

Concave Down: $(0, 2)$

Inflection Pts: $(0, 0)$ and $(2, -16)$

d)



Remember the graph is $f'(x)$ The DERIVATIVE!

34) a) Where $f'(x) = 0$

$$x = -7, -1, 4, 8$$

b) Where $f'(x)$ goes from (+) to (-)

$$x = -1 \quad x = 8$$

c) When $f'(x)$ is decreasing
 $(-3, 2) \cup (6, \infty)$